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Principal-Agent Theory Model in Dynamic Spectrum Sharing

Yang Liu, Zhikui Chen
School of Software
Dalian University of Technology
Dalian, China 116620
Email: {liuyang}{zkchen}@dlut.edu.cn

Xiaoning Lv
School of Management
Dalian University of Technology
Dalian, China 116023
Email: seekyou77@hotmail.com

***Abstract*—This paper points to the dynamic spectrum sharing problem in cognitive radio network and analyzes the behavior of primary user (PU) and secondary user (SU). The relationship between PU and SU can be treated as principal and agent. Then, a principal-agent theory based two-step game model is proposed with the consideration of cooperation and price coefficient which effect the SU's choice and PU's profit. Further, some incomplete information is assumed as random variables so that certainty equivalent is introduced in the model. Both cooperation and price effect are simulated at the end.**

I. Introduction

Due to the limitation of current wireless communication technology, in order to guarantee the quality of radio link, the operators exploit "light payload" way, which makes the effective utilization of spectrum is quite low. However, with the emergence of new wireless applications and devices, the last decade has witnessed a dramatic consuming of radio spectrum. It already becomes one of the biggest issues for government regulatory bodies, such as the Federal Communications Commission (FCC). In addition, another problem is often discussed that, if the number of users and service payload increase and exceed a balance, the performance of communication will degrade much significantly. Fortunately, cognitive radio technology [1], [2] is considered as a promising approach, enabling wireless devices to utilize the spectrum adaptively and efficiently with a high quality.

In the theory of cognitive radio, the users in radio are divided into primary users (PU) and secondary users (SU). PU has superior right and authority in using specific spectrum. SU is the user who can access the PU's vacant spectrum resources under some certain rules after sensing the spectrum conditions of PU. This way is an amazing approach to increase the spectrum utilization. Now many research is based on microeconomics knowledge such as game theory,etc [3].

Literature [10], [12], [13], [17] studied the competition relationship between PU and SU and proposed non-cooperative game models and Nash equilibriums [4] are achieved. [14], [24] considered the mutual cooperation could enhance the overall profits so they modeled with cooperative games. Because PU and SU could only make one choice simultaneously, the deal might be canceled for conflict. [22], [23] proposed multi-step dynamic game models, which partitioned the behavior of both sides with serial. One could observe the other's action and then made his own scheme. [11], [19], [20] further considered the mutual information is usually incomplete and introduced auction games [21] to solve this problem. Some dynamic game models exploited multi-step game and obtained the equilibrium by recursive convergence, which cost the system much. Then, [8], [16] based on more simple two-step model by treating PU and SU as leader and follower generated Stackelberg model. [25] did not use game theory to model this problem but linked the quality and price of the spectrum together, better quality needed more money. PU made a quality-price contract for SUs and encourage SUs buying better quality resources.

This paper is to establish a principal - agent theory based game model. The remainder of this paper is organized as follow: Section 2 shows the basis of the proposed model. Section 3 presents the detailed idea of the general model. Section 4 further gives a systematic model. Simulations are made in Section 5. Section 6 concludes at the end.

II. Theory and Precondition

A. QoS Mapping

In this paper, the "utility" in economics replaces the system-centric QoS indicators (such as throughput, packet loss rate, power, etc.) to evaluate the user's satisfaction for the operational requirements of the network. The basic idea is to evaluate the effectiveness of the resources (e.g. bandwidth, power, etc.) or performance indicators (e.g. data rate, delay, etc.) mapped to the corresponding utility values, and then evaluate the overall effectiveness of the system. Scheduling based on utility function will be operational latency, throughput and other objective attributes mapping to represent the utility value of user satisfaction, whose objective is to maximize the overall effectiveness of the system. And with in-depth understanding of user behavior, based on the design of the utility function with reasonable objectivity and subjectivity, a reasonable mathematical expression should be established.

B. Principal-Agent

This paper introduces the principal-agent theory [7], because the behavior of PU offering idle spectrum resources to SU, can be abstracted as a principal-agent behavior. PU who owns

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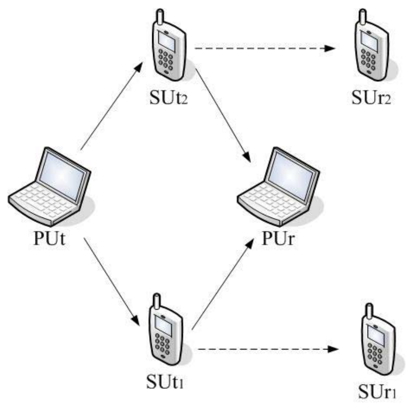

Fig. 1. Cooperative transmit in cognitive radio network

the spectrum is the principal and SU is the agent who needs the resources and wish to buy them. In this paper, if SU wants to get the ways of using PU's spectrum, he has to cooperate with PU. SU can construct a link with PU and then let PU transmit data though this link and simultaneously utilize the holes for his own service, see Fig.1. In the games of cooperation or competition for spectrum resources existing between the principal and agent, or "conflict of interest and coordination" in economics word, we study behavior of the decision-making process for pursuit of maximum effectiveness by the two parties, and establish a feasible mechanisms.

III. General Model Study

A. Some Assumptions

1) H1: We assume that the pricing is quality based [5], [25]. Thus, both principal and agent's benefits is related with the utility of radio link quality. We also have principal is risk neural and agent is risk averse. This assumption conforms to the general principal-agent relationship, which means that the PU's utility function u_1 can be replaced with their monetary gain w_1 (u_1=w_1).While the SU's utility function u_2 fits the risk aversion requirement, characterized by a concave shaped curve (u_2=$-e^{-\rho w_2}$). ρ is the absolute risk aversion degree, w_2 is on behalf of his monetary income.

2) H2: The factors effecting the radio link quality can be classified into two kinds: static and dynamic. We assume the environment of the link between PU and SU is fairly stable at certain time-slot. Hence, we set the effect denoted as θ is a normal random variable, whose mean equates T and variance equates σ^2, $\theta \sim N(T, \sigma^2)$. The dynamic effect is based on the link profit presented by $f(\alpha)$ generated by SU. α is an one-dimensional continuous variable and has no correlation with θ. And the quality based profit (money unit) $Y = f(\alpha)+\theta$, $f'(\alpha) > 0$, which means Y is a increasing function of α. Therefore $Y \sim N(f(\alpha) + T, \sigma^2)$.

3) H3: PU needs to acquire appropriate compensation for his sharing. As H1, the link revenue (S) is relevant to the link quality profit (Y), we have $S = \beta * Y$, $\beta \in [0, 1]$ is cooperation coefficient influenced by PU's ongoing transmission. If $\beta = 0$, it means although PU share his link, SU uses nothing but only take on PU's transmission as a relay. If $\beta = 1$, PU shares all to SU selflessly.

4) H4: PU will provide his idle spectrum with cost (he has paid and got the authority). And the payment function is assumed as follows: $C(\alpha) = b\alpha$, where $b > 0$ indicates pricing coefficient.

From the agent's perspective, the profit is obtained by the spectrum resources supplement. The payment is the influence of the link quality, so his participation constraint must also meet the benefits outweighed the charge. On this basis, the agent's decision-making depends on b set by the principal and β presenting the principals' utilization. According to certain β and b, the agent establishes his profit functions to maximize the utility and clear how much is the optimal amount to buy. When principal is busy (β is low), the incentive way to let agent buy more resources is to decrease the price. If β is high, the principal's rational choice is to have a makeup. Hence, first of all, principal needs to clarify the agent's decision-making mode, then establish the utility function to determine the b with certain β to make his own utility maximized, when the agent is also willing to buy the corresponding spectrum resources.

B. Certainty equivalent

The utility of PU and SU are denoted with Y, which is a random variable, it is hard to measure. So we need to derive the certainty equivalent [6] of two parties' profits. It can be summarized as follows:

$$\begin{aligned} w_2 &= S - C \\ &= \beta(f(\alpha) + \theta) - b\alpha \end{aligned} \tag{1}$$

Because the distribution of θ is normally distributed, so $w_2 \sim N(\omega, \beta^2\sigma^2)$, here

$$\omega = \beta(f(\alpha) + T) - b\alpha. \tag{2}$$

We define $\tilde{w}_2$ as the certainty equivalent [6] of w_2. Since u_2=$-e^{-\rho w_2}$, we have

$$\begin{aligned} E(u_2) &= E(-e^{-\rho w_2}) \\ &= \int_{-\infty}^{\infty} -e^{-\rho w_2} \frac{e^{-\frac{(w_2-\omega)^2}{2\beta^2\sigma^2}}}{\sqrt{2\pi}\beta\sigma} dw_2 \\ &= e^{\frac{\beta^2\sigma^2\rho^2 - 2\rho\omega}{2}} \int_{-\infty}^{\infty} \frac{e^{-\frac{[w_2+(\beta^2\sigma^2\rho)]^2}{2\beta^2\sigma^2}}}{\sqrt{2\pi}\beta\sigma} dw_2 \end{aligned}$$

Where $\int_{-\infty}^{\infty} \frac{e^{-\frac{[w_2+(\beta^2\sigma^2\rho)]^2}{2\beta^2\sigma^2}}}{\sqrt{2\pi}\beta\sigma} dw_2 = 1$. Let $-e^{-\rho\tilde{w}_2} = E(u_2)$, so we have $\tilde{w}_2 = -\frac{\beta^2\sigma^2\rho}{2} + \omega$, then

$$\tilde{w}_2 = \beta(f(\alpha) + T) - b\alpha - \frac{\beta^2\sigma^2\rho}{2} \tag{3}$$

Likewise, PU's monetary income

$$w_1 = Y - S + C = (1 - \beta)Y + b\alpha \tag{4}$$

And u_1=w_1, so the certainty equivalent of w_1 is

$$\tilde{w}_1 = (1-\beta)(f(\alpha)+T) + b\alpha \tag{5}$$

C. practical situation

In practice, SU can only base on some observable results for PU's bill. So the problem can be regarded as a two-step dynamic game. In the first step, SU maximize his own utility $\tilde{w}_2$:

$$max. \quad \beta(f(\alpha)+T) - b\alpha - \frac{\beta^2\sigma^2\rho}{2} \tag{6}$$

The result is:

$$\beta f'(\alpha) = b \tag{7}$$

Suppose α satisfies Eq.7 above.

In the second step, under constraint conditions, PU chooses proper activities to maximize his own utility $\tilde{w}_1$:

$$\begin{aligned} max. \quad & (1-\beta)(f(\alpha)+T) + b\alpha \\ s.t. \quad & \tilde{w}_2 \geqslant w_0 \\ & \beta f'(\alpha) = b \end{aligned} \tag{8}$$

Here w_0 is the lowest benefits with which SU is willing to involve treated as opportunity cost.

IV. Systematic model

In this section we propose a systematic model. For SU's requirement, transmitting rate is one of the most important parameters to evalate the link quality. But $SU_i, i \in \Phi$ needs various transmitting rate for its application which makes the measurement of quality is also different. Therefore, we use satisfactory function [16]–[18] to evaluate it. Here, we use *sigmoid* function [15] as satisfactory function and $R_i, i \in \Phi$ is used to replace the α set in *H2* as transmitting rate for each SU_i. Then, the $f(\alpha)$ can be written with this system model

$$f(R_i) = \frac{w_c}{1+e^{-\zeta_i(Ri-R_{1i})}} \tag{9}$$

Where w_c is monetary coefficient, ζ_i is satisfactory coefficient and R_{1i} is the threshold of protocol requirement. Also the payment $C(\alpha)$ in *H4* is verified to bR_i. Hence, Eq.7 is rewritten:

$$\beta\left(\frac{w_c}{1+e^{-\zeta_i(Ri-R_{1i})}}\right)' = b \tag{10}$$

Satisfying $R_i > R1_i$, we obtain:

$$\begin{aligned} R_i^* = R_{1i} - \tfrac{1}{\zeta_i} ln\tfrac{\beta w_c\zeta_i - 2b - \sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i}}{2b} \\ s.t. \quad 4b \le \beta w_c \zeta_i \end{aligned} \tag{11}$$

The other result $R_i^* = R_{1i} - \frac{1}{\zeta_i} ln\frac{\beta w_c\zeta_i - 2b + \sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i}}{2b}$ is discarded.

proof: if $R_i^* = R_{1i} - \frac{1}{\zeta_i} ln\frac{\beta w_c\zeta_i - 2b + \sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i}}{2b}$ exists, with $R_i^* > R_{1i}$, we have

$$ln\frac{\beta w_c\zeta_i - 2b + \sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i}}{2b} < 0$$

$$\beta w_c\zeta_i - 2b + \sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i} < 2b$$

$$\sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i} < 4b - \beta w_c\zeta_i$$

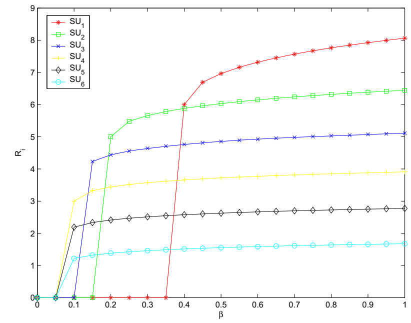

Fig. 2. Relationship between R_i and β of SU

Since $4b - \beta w_c\zeta_i \le 0$, $\sqrt{\beta^2 w_c^2\zeta_i^2 - 4b\beta w_c\zeta_i} < 0$ which is impossible for real value. QED.

$$f(R_i)'' = -\frac{-\zeta_i^2\beta w_c e^{-\zeta_i(R_i-R_{1i})}(1-e^{-\zeta_i(R_i-R_{1i})})}{(1+e^{-\zeta_i(R_i-R_{1i})})^3} \tag{12}$$

Since $R_i > R_{1i}$, we have $e^{-\zeta_i(R_i-R1i)} < 1$. Also because all the coefficient is positive, Eq.12 is negative which says the slope of R_i^* along β decreasing. This just matches the marginal effect in some microeconomics situation.

Next step is to maximize the benefits of PU similarly as Eq.8.

$$\begin{aligned} max. \quad & \textstyle\sum_i^N (1-\beta)\left(\frac{w_c}{1+e^{-\zeta_i(R_i^*-R_{1i})}} + T_i\right) + bR_i^* \\ s.t. \quad & \tilde{w}_2 \geqslant w_0 \end{aligned} \tag{13}$$

V. Simulation

In the simulation, 6 SUs try to access the cognitive spectrum and ζ_i is set to {1,2,3,4,5,6} while R_{1i} is {6,5,4,3,2,1}. $w_c = 1$. w_0 and T are ignored for simplicity.

A. Cooperation coefficient

Here, we set $b = 0.1$ and $\beta \in [0,1]$. Firstly, from Fig.2, we can see six $\beta - R_i$ curves of SU_i. After the cost reaches certain values, these curves have the similar upwards trends, which means SU_i prefers more resources when transmission burden as a relay is low, however the marginal effect decreases. For SU_1, when the cooperation coefficient is larger than 0.4, the resources could be accepted. Similarly, after the β arrives at about 0.2, 0.16, 0.12,0.1,0.1 respectively for $SU_2 \sim SU_6$, the resources can satisfy the requirements. We can also get the situation that who accesses in the spectrum with various β, shown in Fig.4. The long bar means successfully access. Secondly, the rules of PU's benefit based on the cooperation can be achieved, illustrated in Fig.3. The curve is roughly convex which means a maximum exists. When $\beta = 0.41$, the PU get his biggest revenue $u_1^{max} = 5.5362$. According to Fig.4, at this moment, only SU_1 fails in entering. As the vertex passes, due to the much selfless cooperation, the profit goes down.

B. Pricing coefficient

In this part, $\beta = 0.5$ and $b \in [0,1]$ are configured. Fig.5 shows that SU_i buys less spectrum with price appreciate. SU_i

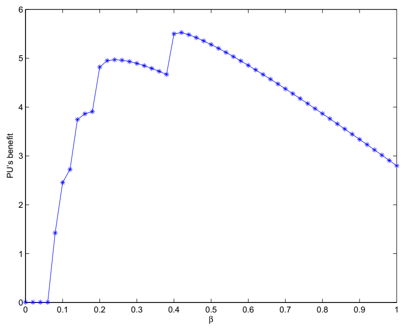

Fig. 3. PU's benefit along β

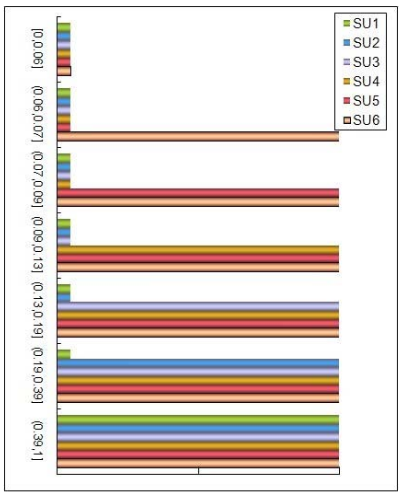

Fig. 4. SUs' access in cooperative effect

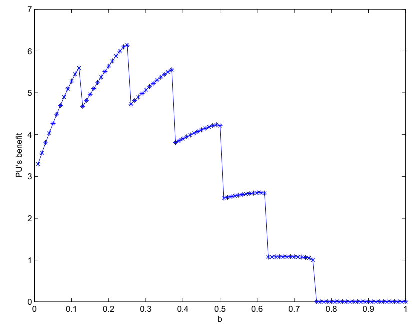

Fig. 6. PU's benefit along b

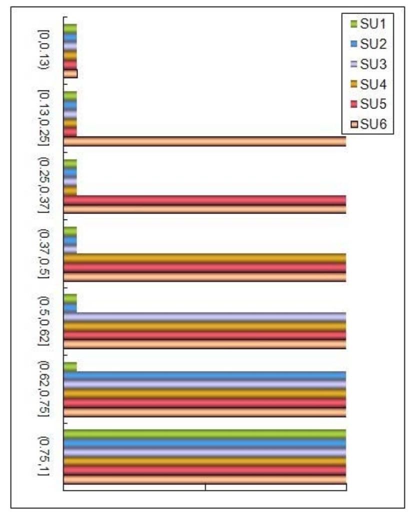

Fig. 7. SUs' access in price effect

with different characters follows different gradient. The curve drops more significant for SU_i Who requires higher rate. Even $SU_1 \sim SU_6$ give up access after the price coefficients reach about 0.13,0.25,0.37,0.50,0.62,0.75 respectively. In Fig.6, the PU's benefits can be divided into six segments, since the number of involvers is changed, also see Fig.7. The forward five segments climb in their region and the last one drops. In the second segment, the profit achieves the maximum, when

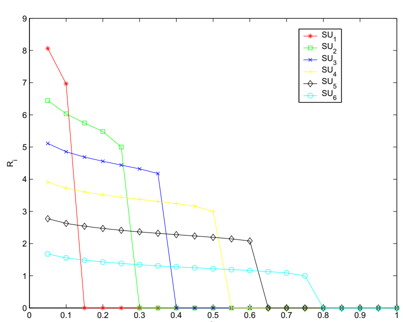

Fig. 5. Relationship between R_i and b of SU

$b = 0.25$, $u_1^{max} = 6.1387$.

VI. Conclusion

In this paper, PU and SU are assimilated to principal and agent for the behaviors of sharing spectrum. And we propose a principal-agent theory based two-step game model. The cooperation and price coefficient are considered in this model which effect the SU's choice and PU's profit. SU generates his own scheme with observed factors in order to guarantee his requirement. Then, PU decides the price setting with SU's choice and maximizes his benefit in certain circumstances. In the simulation step, both cooperation and price effect are examined using proposed model.

References

[1] J. Mitola III, *Cognitive radio: an integrated agent architecture for software defined radio*, Ph.D. Thesis, KTH, 2000.

[2] S. Haykin, *Cognitive radio: brain-empowered wireless communications*,Selected Area in Communication, IEEE Journal on, vol.23(2), pp. 201-220, Feb. 2005.

[3] D. Niyato and E. Hossain, *Microeconomic models for dynamic spectrum management in cognitive radio networks*, invited chapter in Cognitive Wireless Communication Networks, Springer-Verlag, Nov. 2007.

[4] Nash J. *Equilibrium Points in n-person Games*, Proceedings of the National Academy of Science, 1950, (36):48-49.

[5] Hak Ju Kim, *New accounting system for the QoS-enabled multiservice internet: technical overview* , International Journal of Business Information Systems, Jan. 2009, Vol.4(2), pp: 173-194.

[6] Kahneman, Daniel, and Amos Tversky, *Prospect Theory: An Analysis of Decision under Risk*, 1979, Econometrica, 47(2), pp: 263C292.

[7] Bertrand M., Ollainathan S M. *Agents with and without principals* . The American Economic Review, 2000, pp: 203-208.

[8] Chungang Yang, Jiandong Li, *Joint Economical and Technical Consideration of Dynamic Spectrum Sharing: A Multi-Stage Stackelberg Game Perspective*, Vehicular Technology Conference Fall (VTC 2010-Fall), 2010 IEEE 72nd, Ottawa, Sept. 2010.

[9] G. Owen, Game Theory, Academic Press, 1995.

[10] Alyfantis, G.; Marias, G.; Hadjiefthymiades, S.; Merakos, L., *Non-Cooperative Dynamic Spectrum Access for CDMA Networks*, GLOBECOM'07, IEEE, 2007.

[11] Sengupta, S. Chatterjee, M., *An Economic Framework for Dynamic Spectrum Access and Service Pricing*, Networking, IEEE/ACM Transactions on, 2009, Vol.17(4), pp: 1200 - 1213.

[12] Niyato, D. Hossain, E. Zhu Han, *Dynamic spectrum access in IEEE 802.22-based cognitive wireless networks: a game theoretic model for competitive spectrum bidding and pricing*, Wireless Communications, IEEE, 2009, Vol.16(2): 16 - 23

[13] Niyato, D. Hossain, E. Zhu Han, *Competitive spectrum sharing in cognitive radio networks: a dynamic game approach*, Wireless Communications, IEEE Transactions on, 2008, Vol.7(7), pp: 2651 - 2660.

[14] Z. Han, Z. Ji, and K. J. Ray Liu, *Fair Multiuser Channel Allocation for OFDMA Networks Using Nash Bargaining and Coalitions*, Communications, IEEE Transaction on, 2005, Vol.53, pp:1366-1375.

[15] D. H. von Seggern. *CRC Standard Curves and Surfaces with Mathematica*, Second Edition, Chapman & Hall/CRC, 2006.

[16] Jin Zhang and Qian Zhang, *Stackelberg game for utility-based cooperative cognitive radio networks*, in Proc. ACM MobiHoc'09, 2009, pp. 23-32.

[17] H. Lin, M. Chatterjee, S. K. Das, and K. Basu. *Arc: An integrated admission and rate control framework for competitive wireless cdma data networks using noncooperative games*. IEEE Transactions on Mobile Computing, 2005, Vol.4(3), pp:243-258.

[18] G. Stamoulis, D. Kalopsikakis, and A. Kyrikoglou. *Efficient agent-based negotiation for telecommunications services*. GLOBECOM '99, IEEE, 1999, Vol.3, pp:1989-1996.

[19] Xinbing Wang, Zheng Li, etc, *Spectrum Sharing in Cognitive Radio Networks-An Auction-Based Approach*, Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on, 2009, Vol.40(3), pp: 587 - 596.

[20] Yongle Wu, Beibei Wang, K.J.R. Liu T.C. Clancy, *A scalable collusion-resistant multi-winner cognitive spectrum auction game*, Communications, IEEE Transactions on, 2009, Vol.57(12), pp: 3805 - 3816.

[21] V. Krishna, *Auction Theory*, Academic Press, 2002.

[22] Zhu Ji, K.J.R.Liu, *Multi-Stage Pricing Game for Collusion-Resistant Dynamic Spectrum Allocation*, Selected Areas in Communications, IEEE Journal on, 2008, Vol.26(1), pp: 182 - 191.

[23] Niyato, D. Hossain, E., *Competitive Pricing for Spectrum Sharing in Cognitive Radio Networks: Dynamic Game, Inefficiency of Nash Equilibrium, and Collusion*, Selected Areas in Communications, IEEE Journal on, 2008, Vol.26(1), pp: 192 - 202.

[24] Gatsis, N. Marques, A.G. Giannakis, G.B., *Power control for cooperative dynamic spectrum access networks with diverse QoS constraints*, Communications, IEEE Transactions on, 2010, Vol.58(3), pp: 933 - 944.

[25] Lin Gao, Xinbing Wang, Youyun Xu and Qian Zhang,*Spectrum Trading in Cognitive Radio Networks: A Contract-Theoretic Modeling Approach*,Selected areas in communications, IEEE Journal on, 2011, Vol.29(4).